A mechanistic model of an upper bound on oceanic carbon export as a function of mixed layer depth and temperature

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Abstract.
Export production reflects the amount of organic matter transferred from the ocean surface to depth through biological processes. This export is in large part controlled by nutrient and light availability, which are conditioned by mixed layer depth (MLD). In this study, building on Sverdrup’s critical depth hypothesis, we derive a mechanistic model of an upper bound on carbon export based on the metabolic balance between photosynthesis and respiration as a function of MLD and temperature. We find that the upper bound is a positively skewed bell-shaped function of MLD. Specifically, the upper bound increases with deepening mixed layers down to a critical depth, beyond which a long tail of decreasing carbon export is associated with increasing heterotrophic activity and decreasing light availability. We also show that in cold regions the upper bound on carbon export decreases with increasing temperature when mixed layers are deep, but increases with temperature when mixed layers are shallow. A meta-analysis shows that our model envelopes field estimates of carbon export from the mixed layer. When compared to satellite export production estimates, our model indicates that export production in some regions of the Southern Ocean, particularly the subantarctic zone, is likely limited by light for a significant portion of the growing season.

1 Introduction

Photosynthesis in excess of respiration at the ocean surface leads to the production of organic matter, part of which is transported to the deep ocean through sinking and mixing (Volk and Hoffert, 1985). This biological process, known as export production (also called the soft-tissue biological carbon pump) lowers carbon dioxide (CO$_2$) concentrations at the ocean surface and facilitates the flux of CO$_2$ from the atmosphere into the ocean (Falkowski et al., 1998; Ito and Follows, 2005; Sigman and Boyle, 2000).

Export production is frequently assumed to be a function of net community production (NCP), which is defined as the balance between net primary production (NPP) and heterotrophic respiration (HR) or the difference between gross primary production (GPP) and community respiration (CR; HR plus autotrophic respiration, AR; the abbreviations used in this study are presented in Table A; Li and Cassar, 2016):

\[
\text{CO}_2 + \text{H}_2\text{O} \xrightarrow{\text{GPP}} \text{Organic matter} + \text{O}_2, \quad (1)
\]

Export production = NCP – MLD \cdot \frac{d(\text{POC} + \text{DOC})}{dt}, \quad (2)

where POC, DOC, and MLD represent particulate organic carbon, dissolved organic carbon, and mixed layer depth, respectively. If the organic carbon inventory (POC + DOC) in the mixed layer is at steady state, NCP is equal to export production (Eq. 2). Without allochthonous sources of organic matter, if the organic matter inventory in the mixed layer decreases, NCP will be predicted to be transiently smaller than export production. Conversely, export may lag NPP (Henson et al., 2015; Stange et al., 2017), in which case NCP is expected to be greater than export production.

Net community production is in large part regulated by the availability of nutrients and light. Light availability exponentially decays with depth due to absorption by water and its constituents. The mixing of phytoplankton to depth therefore impacts phytoplankton physiology and productivity (Cullen and Lewis, 1988; Lewis et al., 1984), with the
depth-integrated NPP expected to increase down to the euphotic depth. Respiration, however, is often modeled to be some function of organic matter concentration, which is expected to be constant with depth if homogeneously mixed within the mixed layer. Temperature is also believed to be an important control on carbon export because respiration is more temperature sensitive than photosynthesis (Laws et al., 2000; López-Urrutia et al., 2006; Rivkin and Legendre, 2001). Field observations confirm that NCP is generally lower at high temperatures and consistently low when mixed layers are deep. These patterns have been attributed to the balance between depth-integrated photosynthesis (controlled by the availability of nutrients and light) and respiration as a function of MLD and temperature (Cassar et al., 2011; Eveleth et al., 2017; Huang et al., 2012; Shadwick et al., 2015; Tortell et al., 2015). However, descriptions of the underlying mechanisms remain qualitative. Likewise, the effects of light and nutrients on carbon fluxes are difficult to disentangle. For example, high-nutrient, low-chlorophyll regimes in the Southern Ocean have been attributed to iron limitation (Boyd et al., 2000), deep mixed layers, and light limitation (Nelson and Smith, 1991; Mitchell and Holm-Hansen, 1991; Mitchell et al., 1991), or both (Sunda and Huntsman, 1997). To decompose the influence of light and nutrient availability on NCP, we define the upper bound on carbon export from the mixed layer (NCP*) as the maximum export achievable should all limiting factors other than light (taking into account self-shading) be alleviated.

In his seminal paper, Sverdrup presented an elegant model to demonstrate that vernal phytoplankton blooms (i.e., organic matter accumulation at the ocean surface) may be driven by increased light availability when the MLD shoals above a critical depth (Zc; Sverdrup, 1953). In our study, we build upon Sverdrup (1953) and derive a mechanistic model of an upper bound on carbon export based on the metabolic balance of photosynthesis and respiration in the oceanic mixed layer, in which the metabolic balance is derived from MLD, temperature, photosynthetically active radiation (PAR), phytoplankton maximum growth rate (\( \mu_{\text{max}} \)), and heterotrophic activity. Our approach is analogous to other efforts in which mechanistic models were derived to predict proxies for carbon export (e.g., Dunne et al., 2005 and Cael and Follows, 2016). We compare our NCP* model to observations and use this model in conjunction with satellite export production estimates to identify regions in the world’s oceans where light may limit export production. Our key findings are that (1) using parameters available in the literature, the modeled upper bound envelopes field observations of O2/Ar-derived NCP and export production derived from \(^{234}\)Th and sediment traps, and (2) the model identifies regions of the Southern Ocean where carbon export is likely limited by light during part of the growing season.

**2 Model description and comparison to observations**

**2.1 Net community production and light availability**

A conceptual representation of the metabolic balance between volumetric NCP, NPP, and HR profiles is presented in Fig. 1a. According to Eq. (1), the volumetric NCP flux at a given depth (z) in the mixed layer results from the difference between volumetric NPP and HR:

\[
\text{NCP}(z) = \text{NPP}(z) - \text{HR}(z),
\]

where z increases with depth. NPP(z) is a function of the autotroph intrinsic growth rate (\( \mu \)) times the biomass concentration (C). Assuming that the effect of nutrients and light on photosynthetic rates abides by Michaelis–Menten kinetics and neglecting the effect of photoinhibition (Dutkiewicz et al., 2001; Huisman and Weissing, 1994), NPP(z) may be expressed as follows:

\[
\text{NPP}(z) = \mu(z) \cdot C = \frac{N}{N + k_N^m} \cdot \frac{I(z)}{I(z) + k_I^m} \cdot \mu_{\text{max}} \cdot C,
\]

where \( \mu_{\text{max}} \) is the maximum intrinsic growth rate of the autotrophic community, \( N \) and \( k_N^m \) represent the nutrient concentration and half-saturation constant, respectively, and \( I \) and \( k_I^m \) represent the irradiance level and half-saturation constant, respectively; \( \mu_{\text{max}}, N, k_N^m, k_I^m, \) and \( C \) are assumed to be constant or uniform within the mixed layer. The first two terms on the right-hand side of Eq. (4) account for the effect of nutrient and light availability on autotrophic growth rates, and they are hereafter defined as follows for simplicity:

\[
N_m = \frac{N}{N + k_N^m},
\]

\[
I_m(z) = \frac{I(z)}{I(z) + k_I^m}.
\]
\(I(z)\) is modeled as an exponential decay of PAR just beneath the water surface \((I_0)\):
\[
I(z) = I_0 \cdot e^{-K_1 z}, \tag{6}
\]
where \(K_1\) is the light attenuation coefficient, which is assumed to be independent of depth in the mixed layer.

As a first approximation, we assume that \(HR(z)\) is proportional to \(C\) as in previous studies (Dutkiewicz et al., 2001; Huisman and Weissing, 1994; Rivkin and Legendre, 2001; Sverdrup, 1953; White et al., 1991):
\[
HR(z) = r_{HR} \cdot C, \tag{7}
\]
where \(r_{HR}\) represents the intrinsic heterotrophic respiration rate, which is assumed to be dependent on temperature (see below) and independent of depth. In reality, \(HR(z)\) is likely best modeled as a function of the concentration of labile organic matter—an additional term could be included to account for the relationship of total labile organic matter to \(C\).

NCP integrated over the mixed layer (NCP(0,MLD)) can be derived from Eqs. (3)–(7):
\[
NCP(0, MLD) = NPP(0, MLD) - HR(0, MLD) = \int_0^{MLD} NPP(z) dz - \int_0^{MLD} HR(z) dz = N_m \cdot I_m(0, MLD) \cdot \mu_{max} \cdot C - r_{HR} \cdot MLD \cdot C. \tag{8}
\]
The first term on the right side of Eq. (8) represents NPP integrated over the mixed layer (NPP(0,MLD)), which is equivalent to the product of \(\int_0^{MLD} \mu(z) dz\) and \(C\), the former term is modeled to be a function of \(\mu_{max}\) conditioned by nutrient and light availability within the mixed layer. \(I_m(0, MLD)\) can be derived as follows:
\[
I_m(0, MLD) = \int_0^{MLD} I_m(z) dz
= -\frac{1}{K_1} \cdot \ln \left( \frac{I_0 \cdot e^{-K_1 MLD} + k_1^{1}}{I_0 + k_1^{1}} \right). \tag{9}
\]

NCP integrated over the mixed layer (Eq. 8) is a bell-shaped function of MLD as depicted in the schematic diagram in Fig. 1b.

### 2.2 Net community production and phytoplankton biomass concentration

As can be seen from Eq. (8), NCP(0,MLD) is a direct function of \(C\) because NPP(0,MLD) and HR(0,MLD) are proportional to \(C\). NCP(0,MLD) is also an indirect function of \(C\) due to its effect on light attenuation (i.e., \(K_1\)). The attenuation coefficient \(K_1\) can be divided into water and non-water components \((K_1 = K_1^w + K_1^{nw}\); Baker and Smith, 1982; Smith and Baker, 1978a, b), where \(K_1^{nw}\) is controlled by the concentrations of phytoplankton, colored dissolved organic matter (CDOM), and non-algal particles (NAP). In the open ocean where CDOM and NAP covary with phytoplankton (Morel and Priour, 1977), \(K_1\) can be related to \(C\) as follows:
\[
K_1 = K_1^w + k_c \cdot C, \tag{10}
\]
where \(k_c\) is a function of the solar zenith angle, the specific absorption and backscattering coefficients of phytoplankton, and the relationship between phytoplankton, CDOM, and NAP. Because pure water and phytoplankton attenuate light, \(K_1^w\) and \(k_c\) should be greater than zero.

To calculate how NCP(0,MLD) varies as a function of \(C\), we examine its first \(\frac{dNCP(0, MLD)}{dC}\) and second \(\frac{d^2NCP(0, MLD)}{dC^2}\) derivatives with respect to \(C\) based on Eqs. (8) and (10):
\[
\frac{dNCP(0, MLD)}{dC} = N_m \cdot \mu_{max} \cdot \frac{K_1^w \cdot I_m(MLD) + k_c \cdot C \cdot MLD \cdot I_m(MLD)}{K_1^w + k_c \cdot C} - r_{HR} \cdot MLD, \tag{11}
\]
\[
\frac{d^2NCP(0, MLD)}{dC^2} = N_m \cdot k_c \cdot \frac{\mu_{max} \cdot \frac{K_1^w}{K_1} \cdot (MLD \cdot I_m(MLD) - I_m(0, MLD))}{I_0 \cdot e^{-K_1 MLD}} - \frac{k_c \cdot C \cdot I_m(MLD)^2 \cdot MLD^2 \cdot k_1^{1}}{I_0 \cdot e^{-K_1 MLD}} \tag{12}
\]

when MLD > 0, \(I_m(0, MLD) > MLD \cdot I_m(0, MLD)\):
\[
I_m(0, MLD) = \int_0^{MLD} I_0 \cdot e^{-K_1 z} dz
> \int_0^{MLD} I_0 \cdot e^{-K_1 MLD} \cdot MLD^{1} \cdot k_1^{1} dz = MLD \cdot I_m(MLD). \tag{13}
\]

The detailed derivation of Eqs. (11)–(12) can be found in the Supplement. Substituting the inequality (13) into Eq. (12) gives \(\frac{d^2NCP(0, MLD)}{dC^2} < 0\), which suggests that \(\frac{dNCP(0, MLD)}{dC}\) decreases with increasing \(C\). Because increasing \(C\) decreases light availability due to shelf-shading, NPP(0,MLD) saturates with increasing \(C\). Thus, NCP(0,MLD) will reach an asymptote of \(\lim_{C \to \infty} \left( \frac{dNCP(0, MLD)}{dC} \right) = -r_{HR} \cdot MLD < 0\) because \(HR(0, MLD)\) linearly increases with increasing \(C\), while NPP(0,MLD) plateaus (Fig. 2). Additionally, because NCP(0,MLD) must be nil when there is no autotrophic biomass \((NCP(0, MLD)|_{C=0} = 0\), \(\lim_{C \to 0} \left( \frac{dNCP(0, MLD)}{dC} \right) \) must be greater than zero; otherwise the ecosystem would be net heterotrophic, which is unachievable without an allochthonous source of organic matter. \(\lim_{C \to \infty} \left( \frac{dNCP(0, MLD)}{dC} \right) < 0\) and \(\lim_{C \to \infty} \left( \frac{dNCP(0, MLD)}{dC} \right) = -r_{HR} \cdot MLD < 0\) suggest the existence of \(\frac{dNCP(0, MLD)}{dC} \mid_{C=C^*} = 0\).
The dependence of NCP(0, MLD) on C can be conceptually understood in the following way. Given a water column with sufficient nutrients, the critical depth \( Z_c \) and compensation depth \( Z_p \) are expected to shoal as \( C \) increases. When \( C \) is low, NCP(0, MLD) increases with \( C \) because of its greater impact on NPP(0, MLD) than on HR(0, MLD). As \( C \) further increases, the increase in NPP(0, MLD) with \( C \) slows because of light attenuation (i.e., \( K_i \)). There is therefore a \( C^* \) that maximizes the difference between NPP(0, MLD) and HR(0, MLD) leading to NCP(0, MLD) with \( C \) (Fig. 2). Beyond this point (\( C^* \)), further increasing \( C \) will cause self-shading and limit photosynthesis in the deep part of the mixed layer as a result decreasing NCP(0, MLD). Beyond a critical biomass (\( C_c \)), the ecosystem becomes net heterotrophic. Without an allochthonous source of organic carbon, this is only transiently sustainable.

### 2.3 Mixed layer depth and compensation depth

By definition, if NCP(MLD) is less than zero (i.e., net heterotrophy at the bottom of the mixed layer), the MLD must be deeper than \( Z_p \) (MLD > \( Z_p \)) and vice versa. To determine the sign of NCP(MLD), we substitute inequality (13) into Eq. (11). According to the inequality presented in Eq. (13),

\[
K_i I_m (0, \text{MLD}) + k_C C MLD I_m (\text{MLD})
\]

in Eq. (11) must be greater than

\[
\frac{K_i^* \text{MLD} I_m (\text{MLD}) + k_C C MLD I_m (\text{MLD})}{K_i^* + k_C C MLD I_m (\text{MLD})},
\]

which is equal to

\[
\text{MLD} \cdot I_m (\text{MLD}).
\]

After simple rearrangements, the substitution of inequality (13) into Eq. (11) leads to

\[
\frac{dNCP(0, \text{MLD})}{dC} > \text{MLD} \cdot \frac{(N_m \cdot I_m (\text{MLD}) \cdot \mu_{\text{max}} - r_{HR})}{(N_m \cdot I_m (\text{MLD}) \cdot \mu_{\text{max}} - r_{HR})}.
\]

### 2.4 An upper bound on carbon export

Equations (11)–(13) delineate the conditions for an upper bound on carbon export (NCP*). In order to simplify the relationship of NCP* to MLD and temperature, we approximate
I_m(0, MLD):

\[ I_m(0, MLD) = -\frac{1}{K_1} \ln \left( 1 + \frac{I_0}{I_0 + K_m} \cdot e^{-K_1 \cdot MLD - 1} \right) \]

\[ \approx -\frac{1}{K_1} \ln (1 - I_m(0)), \]  \hspace{1cm} (15)

where \( I_m(0) = \frac{K_0}{I_0 + K_m} \). Based on Eq. (15), NCP(0, MLD) in Eq. (8) can be approximated as

\[ \text{NCP}(0, \text{MLD}) = C \cdot \text{MLD} \cdot \left( \frac{1}{K_1 \cdot \text{MLD}} \cdot \mu^* - r_{HR} \right), \]  \hspace{1cm} (16)

where \( \mu^* = -\ln (1 - I_m(0)) \cdot N_m \cdot \mu_{\text{max}} \). To evaluate the approximation accuracy of Eq. (15), we compare the upper bound for the approximated model with the maximum growth rate and heterotrophic respiration ratio for \( T = 0^\circ \text{C} \), respectively. \( P_t \) is commonly assumed to equal 0.0663 (Eppley, 1972). Substituting Eqs. (20a) and (20b) into Eq. (19) yields

\[ \sqrt{\text{NCP}}^* = a_2 \cdot \sqrt{-\ln (1 - I_m(0))} + a_3 \cdot \sqrt{P_t \cdot T \cdot \text{MLD}}, \]  \hspace{1cm} (21)

where \( a_1 = -\sqrt{K_w \cdot r_{HR} / k_c} \) and \( a_2 = \sqrt{N_w \cdot \mu_{\text{max}} / k_c} \). Constants \( a_1 \) and \( a_2 \) are functions of \( r_{HR} \) and \( \mu_{\text{max}} \), respectively, which are generally modeled to increase with temperature \( (T) \) (Eppley, 1972; Rivkin and Legendre, 2001):

\[ \mu_{\text{max}} = \mu_{\text{max}}^0 \cdot e^{P_t \cdot T}, \]  \hspace{1cm} (20a)

\[ r_{HR} = r_{HR}^0 \cdot e^{B_t \cdot T}, \]  \hspace{1cm} (20b)

Figure 3. Upper bounds derived using the original and approximated models. The upper bound for the original model (Eqs. 8–10) is estimated through a nonlinear optimization approach. The upper bound for the approximated model is calculated analytically from Eq. (19). The models use the constants listed in Table 1 and \( I_m(0) = 0.9 \). Decreasing \( I_m(0) \) and increasing \( r_{HR} \) results in greater discrepancies between the original and approximated models in regions with shallow mixed layers.
Figure 4. Modeled upper bound on carbon export production compared to field observations as a function of mixed layer depth (MLD) and sea surface temperature (SST). (a) The thick gray line represents the upper bound fitted to the net community production (NCP) data. Dashed lines represent the upper bounds calculated using parameters available in the literature (Table 1). (b) NCP as a function of SST with isopleths of constant upper bounds color coded for MLD. NCP observations are color coded with MLD. (c) Surface representing the envelope of the modeled upper bound of carbon export production as a function of SST and MLD. Bars represent field observations color coded with the ratio of NCP to the upper bound. Observations are based on $^{234}$Th and sediment trap estimates of carbon export production and $O_2$/Ar-derived NCP. A stoichiometric ratio of $O_2$/C = 1.4 was used to convert NCP from $O_2$ to C units (Laws, 1991). To account for the effect of PAR on export production, both MLD and carbon fluxes are normalized to $-\log(1 - I_m(0))$ (see Eqs. 19 and 21). The temperature dependence of $r_{HR}$ was modeled as $r_{HR} = 0.08 \cdot T_{\text{max}}$.

2.5.2 Results and discussion

Overall, we find that NCP* calculated using published parameters (Table 1) does a good job of enveloping the carbon export observations reported in the literature (Fig. 4a). Samples on the NCP* envelope (upper bound) are likely regulated by light availability. Conversely, points below the upper bound may be nutrient limited. As expected, NCP* increases with $\mu_{\text{max}}$ and decreases with $r_{HR}$. Model parameters $a_1 = -1.78$ and $a_2 = 14.75$ (Eq. 19) provide the best fit to the upper bound of $O_2$/Ar NCP as a function of MLD. When compared to parameters available in the literature (Table 1), we find the best fit to our modeled upper bound when using $\mu_{\text{max}}$ and $r_{HR}$ of 1.2 and 0.2 $\text{d}^{-1}$, respectively. When accounting for the effect of $T$ on $\mu_{\text{max}}$ and $r_{HR}$, model constants $a_2 = -1.53$ and $a_4 = 13.39$ (Eq. 21) best fit the upper bound on $O_2$/Ar NCP, SST, and MLD observations.

Our results show that NCP* decreases with increasing MLD in warmer waters (Fig. 4b and c) because the temperature effect contributes to part of the relationship between export production and MLD in Fig. 4a. Interestingly, NCP* increases with $T$ in colder waters and shallow mixed layers (Fig. 4c). This is because NCP* reflects the balance between productivity ($a_4 \cdot \sqrt{e^{I_m(T)}}, \sqrt{-\ln(1 - I_m(0))}$) and heterotrophic respiration ($a_3 \cdot \sqrt{e^{B_t(T)}}, \sqrt{\text{MLD}}$). In a shallow, cold mixed layer, the change in productivity with $T$ ($\frac{d}{dT} (a_4 \cdot \sqrt{e^{I_m(T)}}, \sqrt{-\ln(1 - I_m(0))})$) is greater than that of heterotrophic respiration ($\frac{d}{dT} (a_3 \cdot \sqrt{e^{B_t(T)}}, \sqrt{\text{MLD}})$). These results could explain part of the variability in the relationship...
between NCP and SST reported in previous studies (Li and Cassar, 2016). Our NCP* model does not perform as well in warmer, deep mixed layers where high variability in export ratio maxima have also been reported (Cael and Follows, 2016). This may stem from uncertainties in observations, the differing relationship between $T$, $\mu_{\text{max}}$, and $r_{\text{HR}}$ at high temperature, and/or violations of our assumptions (see the “Caveats and limitations” section).

Several recent studies have explored the relationship of NCP to oceanic parameters based on various statistical approaches (Cassar et al., 2015; Chang et al., 2014; Huang et al., 2016). This may stem from uncertainties in observations, the differing relationship between $T$, $\mu_{\text{max}}$, and $r_{\text{HR}}$ at high temperature, and/or violations of our assumptions (see the “Caveats and limitations” section).

Recent studies have explored the relationship of NCP to oceanic parameters based on various statistical approaches (Cassar et al., 2015; Chang et al., 2014; Huang et al., 2012; Li and Cassar, 2016; Li et al., 2016). Our model can shed some light onto the mechanisms driving some of these patterns. To that end, we substitute Eq. (9) into Eq. (8):

$$\text{NCP}(0, \text{MLD}) = C \cdot \text{MLD} \left( -\frac{N_m \cdot \mu_{\text{max}}}{K_1 \cdot \text{MLD}} \ln \left( \frac{I_0 \cdot e^{-K_1 \cdot \text{MLD} + k_1 I_m}}{I_0 + k_1 I_m} \right) - r_{\text{HR}} \right).$$  \hspace{1cm} (22)

Rearranging Eq. (22) results in

$$\text{NCP}_B = \frac{\text{NCP}(0, \text{MLD})}{C \cdot \text{MLD}} = -\ln \left( \frac{I_0 \cdot e^{-K_1 \cdot \text{MLD} + k_1 I_m}}{I_0 + k_1 I_m} \right) + \frac{N_m \cdot \mu_{\text{max}}}{C \cdot \text{MLD} \cdot \text{PAR}_{\text{ML}}} \cdot r_{\text{HR}},$$  \hspace{1cm} (23)

where $\text{NCP}_B$ is the biomass-normalized volumetric NCP, $\text{PAR}_{\text{ML}}$ is the average PAR in the mixed layer ($\text{PAR}_{\text{ML}} = \frac{1}{K_1 \cdot \text{MLD}} \cdot I_0$), and $-\ln \left( \frac{I_0 \cdot e^{-K_1 \cdot \text{MLD} + k_1 I_m}}{I_0 + k_1 I_m} \right) + \frac{N_m \cdot \mu_{\text{max}}}{C \cdot \text{MLD} \cdot \text{PAR}_{\text{ML}}} \cdot r_{\text{HR}}$ correspond to the slope and offset, respectively. The scatter in the relationship between chlorophyll-normalized volumetric NCP and $\text{PAR}_{\text{ML}}$, as reported in previous studies (Bender et al., 2016), can likely be explained by the effect of temperature and the availability of nutrients and light (among other properties) on the slope and offset of Eq. (23). Equation (22) can also be reorganized to assess how environmental conditions may impact the export ratio (ef):

$$ef = \frac{\text{NCP}(0, \text{MLD})}{\text{NPP}(0, \text{MLD})} = 1 - \frac{K_1 \cdot \text{MLD}}{-\ln \left( \frac{I_0 \cdot e^{-K_1 \cdot \text{MLD} + k_1 I_m}}{I_0 + k_1 I_m} \right)}$$

where $\frac{1}{N_m \cdot \mu_{\text{max}}} \cdot \frac{r_{\text{HR}}}{K_1}$ is proportional to $e^{(B_T - P_T \cdot T)}$. Equation (24) is consistent with multiple studies that predict decreasing ef with increasing temperature (Cael and Follows, 2016; Dunne et al., 2005; Henson et al., 2011; Laws et al., 2000; Li and Cassar, 2016). In fact, Eq. (5) of Cael and Follows (2016) can easily be derived from Eq. (24) (see the Supplement). Equation (24) also highlights the fact that a multitude of factors may confound the dependence of ef on temperature (including varying MLD, light attenuation, and availability of nutrients and light). This again may explain some of the conflicting observations recently reported in the literature (e.g., Maiti et al., 2013); the effect of temperature may be masked by changes in community composition (Britten et al., 2017; Henson et al., 2015). One therefore needs to account or correct for the multitude of confounding factors when predicting the effect of a given environmental condition (e.g., temperature, mineral ballast, and NPP) on the export ratio.

### 3 Spatial distribution of the upper bound on carbon export

We estimate the global distribution of the upper bound of carbon export using Eq. (19) and climatological monthly MLD and PAR. In general, NCP* is high in low latitudes and low in the North Atlantic and Antarctic Circumpolar Current (ACC) in the Southern Ocean (Fig. 5a). As expected, this spatial pattern is controlled by MLD (see Fig. S1). Satellite-derived estimates of NCP (Li and Cassar, 2016) are approximately 10% of global NCP*, reflecting the high degree of nutrient limitation in the oceans. We also derive a global NCP* map using Eq. (21) and find that the global NCP* estimate is very sensitive to the temperature dependence of $r_{\text{HR}}$. For example, decreasing the $B_T$ in $r_{\text{HR}} = \frac{1}{N_m \cdot \mu_{\text{max}} \cdot \mu_{\text{eff}}} \cdot e^{(B_T - P_T \cdot T)}$ from 0.11 to 0.08 (as used in Rivkin and Legendre, 2001, and López-Urrutia et al., 2006) increases the global NCP* budget by a factor of 2.4. Large differences in NCP* in low latitudes in large part explain this change. In light of the large uncertainties in the relationship between $r_{\text{HR}}$ and $T$ (Cael and Follows, 2016;
López-Urrutia et al., 2006), we hereafter only discuss NCP* estimates derived from Eq. (19).

To estimate how close export production is to its upper bound, we calculate the ratio of export production to NCP* \( f_{pt} \). Low \( f_{pt} \) regimes represent ecosystems likely regulated by nutrient availability (i.e., ecosystems that have not reached their full export potential based on MLD and surface PAR). As expected, low-latitude and subtropical regions have low \( f_{pt} \) (Fig. 5b). High \( f_{pt} \) regimes represent ecosystems that have reached their full light potential and are therefore less likely to respond to nutrient addition because of light limitation (e.g., the North Atlantic and ACC; Fig. 5b). In these regions, especially the subantarctic region, \( f_{pt} \) is high in the spring (Fig. 5c) and decreases in the summer (Fig. 5d), suggesting that export production is likely co-limited by nutrient and light availability. This may in part explain the lower response to iron fertilization in the subantarctic region where substantial increases in surface chlorophyll were only observed in regions with shallower mixed layers (Boyd et al., 2007, 2000; de Baar et al., 2005).

Also shown in Fig. 5 are the biological pump efficiency and export ratios (Fig. 5e and f, respectively). These various proxies reflect different components of the biological pump. Whereas \( f_{pt} \) reflects the export potential based on current MLD and light availability, the biological pump efficiency reflects the potential as derived from nutrient distribution in the oceans and estimated from the extent of nutrient removal from the surface ocean (Sarmiento and Gruber, 2006) or the proportion of regenerated nutrients at depth (Ito and Follows, 2005). A revised estimate of the global biological pump efficiency, estimated based on the proportion of regenerated to total nutrients (preformed + regenerated) at depth, is around 30–35% (Duteil et al., 2013). The ef ratio, however, describes how much of production is exported as opposed to recycled in the surface (Dunne et al., 2005). Ultra-oligotrophic subtropical waters have a low export ratio and a strong biological pump efficiency with exhaustion of nutrients at the ocean surface; they have therefore not reached their full light potential (low \( f_{pt} \)) because of the strong stratification and nutrient limitation. The seasonal pattern of \( f_{pt} \) in the subantarctic region suggests that low biological pump efficiency is the result of light limitation in the austral spring and nutrient (likely Fe) and light limitation in the austral summer.

4 Caveats and limitations

There are a multitude of uncertainties, simplifications, and approximations in our model and field observations. Among others, these include the following.
In our study, we used a model that builds on Sverdrup’s critical depth hypothesis. There are competing hypotheses to explain phytoplankton bloom phenology (timing and intensity), including the “coupling hypothesis” or “disturbance recovery hypothesis” (Behrenfeld, 2010; Boss and Behrenfeld, 2010) and the “critical turbulence hypothesis” (Brody and Lozier, 2015; Huisman et al., 1999; Taylor and Ferrari, 2011). In the case of top-down control, any respiratory grazing loss not accounted for by our loss term would behave as a system not reaching its full light potential (NCP*). Conversely, any grazing loss associated with export (e.g., rapidly sinking fecal pellets and other zooplankton-mediated export pathways) would minimize respiratory losses, thereby bringing NCP closer to its upper bound based on light availability. These opposing effects are beyond the scope of this study but could be modeled, especially as we learn more about their impacts on carbon fluxes through new efforts such as the NASA EXPORTS program (Siegel et al., 2016). See also the point below on mixing vs. mixed layer depth.

- Phytoplankton biomass concentration ($C$) may vary with depth in the mixed layer, especially for water columns experiencing varying degrees of turbulent mixing. In addition, MLD is not always the best proxy for light availability with the mixing layer in some cases deviating from the mixed layer (Franks, 2015; Huisman et al., 1999). The factors defining the MLD also vary in different oceanic regions.

- For simplicity, we model the dependence of photosynthesis on irradiance assuming Michaelis–Menten kinetics, which does not account for photoinhibition. More accurate models can be found in other studies (Platt et al., 1980). Due to optional absorption, $K_1$ also varies with depth in the mixed layer. Additionally, the linear relationship between $K_1$ and $C$ is influenced by CDOM, NAP, and other environmental factors (e.g., solar zenith angle; Gordon, 1989).

- The $\mu_{\text{max}}$ and $r_{HR}$ values are influenced by environmental factors other than temperature, including community structure (Chen and Laws, 2017), and may vary with depth within the mixed layer (Smetacek and Passow, 1990). For these reasons, the equations relating $\mu_{\text{max}}$ and $r_{HR}$ (i.e., $B_1$ and $P_1$) to temperature also carry significant uncertainties (Bisninger et al., 2008; Edwards et al., 2016; Kremer et al., 2017; López-Urrutia and Morán, 2007; Rivkin and Legendre, 2001), which impacts our estimates of the upper bound on carbon export, especially in warmer regions. As in other recent studies (Cael and Follows, 2016; Cael et al., 2017; Dutkiewicz et al., 2001; Gong et al., 2015, 2017; Huisman et al., 2006; Taylor and Ferrari, 2011), we model heterotrophic respiration to vary in proportion to phytoplankton concentration. The model could be further improved by explicitly including the concentration of heterotrophs. See the point above on the grazing effect on export with regards to $r_{HR}$.

- NCP may underestimate export production when accompanied by a decrease in the inventory of organic matter in the mixed layer (see the Introduction and Eq. 2).

- Our field observations are limited, mostly focusing on the spring and summer seasons, and harbor significant uncertainties. For example, deep mixed layers can bias the $O_2/Ar$ method low if entrainment of deeper waters brings low $O_2$ into the mixed layer. Descriptions of these uncertainties are presented in other studies (Bender et al., 2011; Cassar et al., 2014; Jonsson et al., 2013).

- Finally, our study is only relevant to the mixed layer. It does not account for productivity below the mixed layer, which can be important in some regions such as the subtropical ocean.

5 Conclusions

In this study, we derived a mechanistic model of an upper bound on carbon export (NCP*) based on the metabolic balance between photosynthesis and respiration of the plankton community. The upper bound is a positively skewed bell-shaped function of mixed layer depth (MLD). At low temperatures, the upper bound decreases with temperature if mixed layers are deep, but increases with temperature if mixed layers are shallow. We used this model to derive a global distribution of an upper bound on carbon export as a function of MLD and surface PAR, which shows high values in low latitudes and low values in high latitudes due to deep MLD. To examine how current export production compares to this upper bound in the world’s oceans, we calculated the ratio of satellite export production estimates to the upper bound derived by our model. High ratios of export production to NCP* in the North Atlantic and ACC indicate that export production in these regions is likely co-limited by nutrient and light availability. Overall, our results may explain the differences in carbon export measured during past iron fertilization experiments (e.g., subantarctic and polar regions), inform future iron fertilization experiments, help in the development of remotely sensed carbon export algorithms, and improve predictions of the response of marine ecosystems to a changing climate.

Data availability. Our $O_2/Ar$ measurements can be downloaded from [https://sites.duke.edu/cassar/remote-sensing-export/](https://sites.duke.edu/cassar/remote-sensing-export/) (Li and Cassar, 2016).
### Appendix A: Model symbols, abbreviations, and units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>Mixed layer depth</td>
<td>m</td>
</tr>
<tr>
<td>$\text{MLD}_{\text{Cmax}}$</td>
<td>Maximum MLD corresponds to maximum achievable autotroph biomass concentration</td>
<td>m</td>
</tr>
<tr>
<td>$z$</td>
<td>Depth</td>
<td>m</td>
</tr>
<tr>
<td>$Z_c$</td>
<td>Critical depth</td>
<td>m</td>
</tr>
<tr>
<td>$Z_p$</td>
<td>Compensation depth</td>
<td>m</td>
</tr>
<tr>
<td>GPP($0, z$)</td>
<td>Gross primary production</td>
<td>mmol C m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>NPP($z$)</td>
<td>Net primary production at depth $z$</td>
<td>mmol C m$^{-3} d^{-1}$</td>
</tr>
<tr>
<td>NPP($0, z$)</td>
<td>Net primary production above depth $z$</td>
<td>mmol C m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>NCP($z$)</td>
<td>Net community production at depth $z$</td>
<td>mmol C m$^{-3} d^{-1}$</td>
</tr>
<tr>
<td>NCP($0, z$)</td>
<td>Net community production above depth $z$</td>
<td>mmol C m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>HR($z$)</td>
<td>Heterotrophic respiration at depth $z$</td>
<td>mmol C m$^{-3} d^{-1}$</td>
</tr>
<tr>
<td>HR($0, z$)</td>
<td>Heterotrophic respiration above depth $z$</td>
<td>mmol C m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>NCP*</td>
<td>The maximum NCP for a given MLD (upper bound on carbon export)</td>
<td>mmol C m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>NCP$_B$</td>
<td>NCP normalized to autotroph biomass inventory in the mixed layer</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$\epsilon f$</td>
<td>Export ratio</td>
<td>unitless</td>
</tr>
<tr>
<td>$\epsilon pt$</td>
<td>Ratio of satellite export production estimates to the upper bound on carbon export</td>
<td>unitless</td>
</tr>
<tr>
<td>N</td>
<td>Nutrient concentration</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$k_N^m$</td>
<td>Half-saturation constant for nutrient concentration</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Nutrient effect on phytoplankton growth $N_m = \frac{N}{N + k_N^m}$</td>
<td>unitless</td>
</tr>
<tr>
<td>PAR</td>
<td>Photosynthetically active radiation</td>
<td>einstein m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>$I_0$</td>
<td>Photosynthetically active radiation just beneath water surface</td>
<td>einstein m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>$I(z)$</td>
<td>Photosynthetically active radiation at depth $z$</td>
<td>einstein m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>$k_l^m$</td>
<td>Half-saturation constant for irradiance</td>
<td>einstein m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>$I_m(z)$</td>
<td>Light effect on phytoplankton growth at depth $z$, $I_m(z) = \frac{I(z)}{I(z)} = \frac{I_0 e^{-k_l^m z}}{I_0 e^{-k_l^m z} + k_l^m}$</td>
<td>unitless</td>
</tr>
<tr>
<td>$I_m(0, z)$</td>
<td>Integrated light effect on phytoplankton growth above depth $z$, $I_m(0, z) = -\frac{1}{k_l^m} \ln \left( \frac{I_0 e^{-k_l^m z} + k_l^m}{I_0 e^{-k_l^m 0} + k_l^m} \right)$</td>
<td>unitless</td>
</tr>
<tr>
<td>PAR$_{ML}$</td>
<td>Average PAR in the mixed layer (PAR$_{ML} = \frac{1-e^{-k_l^m \text{MLD}}}{k_l^m \text{MLD}} \cdot I_0$)</td>
<td>einstein m$^{-2} d^{-1}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Phytoplankton growth rate</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$\mu_{\text{max}}$</td>
<td>Maximum phytoplankton growth rate</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Maximum phytoplankton growth rate for $T = 0{^\circ}$C</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$r_{HR}$</td>
<td>Heterotrophic respiration ratio</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$r_{HR}^*$</td>
<td>Heterotrophic respiration ratio for $T = 0{^\circ}$C</td>
<td>d$^{-1}$</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Light attenuation coefficient ($K_1 = K_1^w + K_1^{nw}$)</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$K_1^w$</td>
<td>Light attenuation coefficient due to water</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$K_1^{nw}$</td>
<td>Light attenuation coefficient due to optically active components</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Specific attenuation coefficient for irradiance</td>
<td>m$^2$ mmol$^{-1}$</td>
</tr>
<tr>
<td>C</td>
<td>Phytoplankton biomass concentration</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$C^*$</td>
<td>Phytoplankton biomass concentration that maximizes NCP</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$C_{\text{max}}$</td>
<td>Maximum achievable autotroph biomass concentration</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>POC</td>
<td>Particulate organic carbon</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>DOC</td>
<td>Dissolved organic carbon</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>CDOM</td>
<td>Colored dissolved organic matter</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>NAP</td>
<td>Non-algal particles</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Temperature dependence for phytoplankton growth rate</td>
<td>°C$^{-1}$</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Temperature dependence for heterotrophic respiration ratio</td>
<td>°C$^{-1}$</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>Carbon dioxide</td>
<td>ppmv</td>
</tr>
</tbody>
</table>
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