Supplmentary Material

This supplmentary material contains descriptions of the ensemble of 54 runs in the Earth system model GENIE used to simulate the uncertainty in estimating remineralisation rates arising from using model circulation rates. Table 1 contains details on the parameters varied, table 2 details differences in overturning circulation and figure 1 shows a comparison of the salinity and temperature fields from each ensemble member with the standard configuration of GENIE.

Context of Transport Matrices in GENIE

The Transport Matrix as described in Khatiwala *et al.,* (2005) and Khatiwala (2007) contains the finite difference tendency calculated in an ocean model:

$$\frac{d\mathbf{c}}{dt} = \frac{\mathbf{c}^{n+1} - \mathbf{c}^{n}}{\Delta t} = \mathbf{A'}^{n}\mathbf{c}^{n} + \mathbf{q'}^{n} \tag{1}$$

where **A**' is the transport matrix containing the finite difference tendencies (dt^{-1}), **c** is a tracer (mol kg^{-1}) and **q**' is a vector of source/sinks (mol kg^{-1} dt^{-1}). The superscripts (n) refer to the time step index. Rearranging Equation (1) for $\mathbf{c}^{n+1}$:

$$\mathbf{c}^{n+1} = (\mathbf{I} + \mathbf{A}'\Delta t)\mathbf{c}^{n} + \mathbf{q'}^{n}\Delta t \tag{2}$$

Equation (2) predicts the tracer field after one timestep ($\mathbf{c}^{n+1}$) from the effect of ocean circulation ($\mathbf{I} + \mathbf{A}'\Delta t$) on the tracer at the previous timestep ($\mathbf{c}^{n}$) plus any sources or sinks for the tracer over the timestep ($\mathbf{q'}^{n}\Delta t$). In GENIE each tracer experiment diagnoses the tracer distribution resulting from the modelled ocean circulation acting on a unit concentration in a single grid-box, i.e., 1 mol kg^{-1}, after one time step. The resulting concentrations form the coefficients in each column. Therefore, the GENIE transport matrix (**A**) described and used in this study is equivalent to ($\mathbf{I} + \mathbf{A}'\Delta t$) from equation (2).

Supplementary Tables and Figures

Table 1: Parameters values from the GENIE ensemble used in this study from Annan et al. (2005). The parameter values for the standard 8-level model in Ridgwell et al. (2007) are also shown.

Parameter	Reference	Min.	Max.	SYN
Ocean				
temp0	go_10	10.115	24.904	0.000
temp1	go_11	10.115	24.904	0.000
rel	go_12	0.9	0.9	0.9
sc_tau	go_13	1.078	2.320	1.932
iso	go_14	2480.917	6496.541	4488.812
dia	go_15	$5.13\text{x}10^{-6}$	$5.14\text{x}10^{-5}$	$2.72\text{x}10^{-5}$
inverse minimum drag	go_16	1.853	4.565	2.940
Atmosphere				
atm. Diff. amp. T	ea_12	3045602	5357693	4667550
atm. Diff. amp. Q	ea_13	1274689	2162105	1097107
dist 'n' width	ea_14	0.923	2.005	1.083
slope	ea_15	$4.89\text{x}10^{-3}$	$1.48\text{x}10^{-1}$	$6.33\text{x}10^{-2}$
T_z	ea_16	$-2.40\text{x}10^{-3}$	$2.30\text{x}10^{-1}$	$1.12\text{x}10^{-1}$
q_z	ea_18	$4.21\text{x}10^{-2}$	$2.30\text{x}10^{-1}$	$2.27\text{x}10^{-1}$
q_m	ea_19	$4.21\text{x}10^{-2}$	$2.30\text{x}10^{-1}$	$2.27\text{x}10^{-1}$
extra1a	ea_25	$-4.29\text{x}10^{-2}$	$-2.60\text{x}10^{-2}$	$2.12\text{x}10^{-2}$
extra1b	ea_26	$1.48\text{x}10^{-1}$	$2.43\text{x}10^{-1}$	$1.20\text{x}10^{-1}$
extra1c	ea_27	$1.56\text{x}10^{-1}$	$2.58\text{x}10^{-1}$	$1.27\text{x}10^{-1}$
Ice				
sea-ice eddy diffusivity	gs_11	2023.114	8000	6200

Table 2: Minimum and maximum overturning circulations (Sv) for each ensemble member from Annan et al. (2005) and for the standard GENIE configuration corresponding to the SYN dataset (Ridgwell et al. 2007) calculated at the end of the 10000 year spin-up.

Ens. Member	Global Min.	Global Max.	Atlantic Min.	Atlantic Max
1	-44.984	44.424	-1.145	20.013
2	-42.595	41.983	-1.009	20.767
3	-48.919	48.096	-0.471	17.1
4	-37.221	36.743	-0.781	17.913
5	-40.909	37.746	-0.965	18.436
6	-44.138	41.438	-0.964	18.519
7	-41.648	39.93	-1.199	20.104
8	-33.106	30.128	-0.773	16.518
9	-37.825	35.532	-0.968	16.697
10	-38.763	38.677	-0.976	17.562
11	-45.308	43.577	-0.918	17.668
12	-43.51	41.387	-1.12	17.875
13	-36.348	32.935	-0.91	16.756
14	-40.963	38.4	-0.918	17.543
15	-35.481	33.514	-0.771	16.994
16	-36.058	35.122	-1.213	19.245

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Ens. Member	Global Min.	Global Max.	Atlantic Min.	Atlantic Max
17	-45.322	40.9	-1.287	18.462
18	-36.093	33.451	-1.123	17.622
19	-43.719	41.425	-0.97	17.7
20	-39.29	36.775	-1.02	18.446
21	-46.585	43.208	-0.946	17.063
22	-46.512	40.972	-1.605	22.607
23	-44.881	41.41	-0.891	16.35
24	-43.417	42.392	-0.888	15.656
25	-41.159	37.946	-0.979	17.759
26	-38.099	36.185	-1.279	18.826
27	-40.831	37.522	-0.907	16.188
28	-36.051	32.139	-1.267	17.629
29	-41.826	39.273	-1.015	18.428
30	-39.999	37.343	-0.883	16.047
31	-44.028	43.078	-1.128	18.89
32	-40.015	38.424	-0.946	17.374
33	-45.535	41.939	-0.975	17.5
34	-37.05	33.947	-0.933	18.752
35	-46.752	43.682	-2.142	19.603
36	-40.286	37.239	-2.18	18.173
37	-44.404	42.344	-0.639	16.259
38	-38.082	37.064	-1.052	16.523
39	-39.32	36.908	-2.129	17.893
40	-34.405	32.641	-1.369	18.476
41	-38.175	38.101	-0.912	19.18
42	-39.147	37.322	-1.099	18.696
43	-45.06	40.854	-1.08	19.603
44	-43.422	42.015	-1.049	18.123
45	-50.089	44.883	-0.997	18.595
46	-40.868	36.177	-0.942	19.541
47	-42.207	39.9	-0.51	15.787
48	-36.276	32.168	-2.061	18.242
49	-41.262	37.757	-0.995	17.64
50	-42.064	39.877	-0.827	16.787
51	-41.28	37.355	-1.025	16.997
52	-39.448	37.092	-1.142	17.869
53	-36.709	32.89	-0.705	15.349
54	-42.012	39.44	-1.308	17.956
SYN	-44.751	40.226	-0.177	18.221

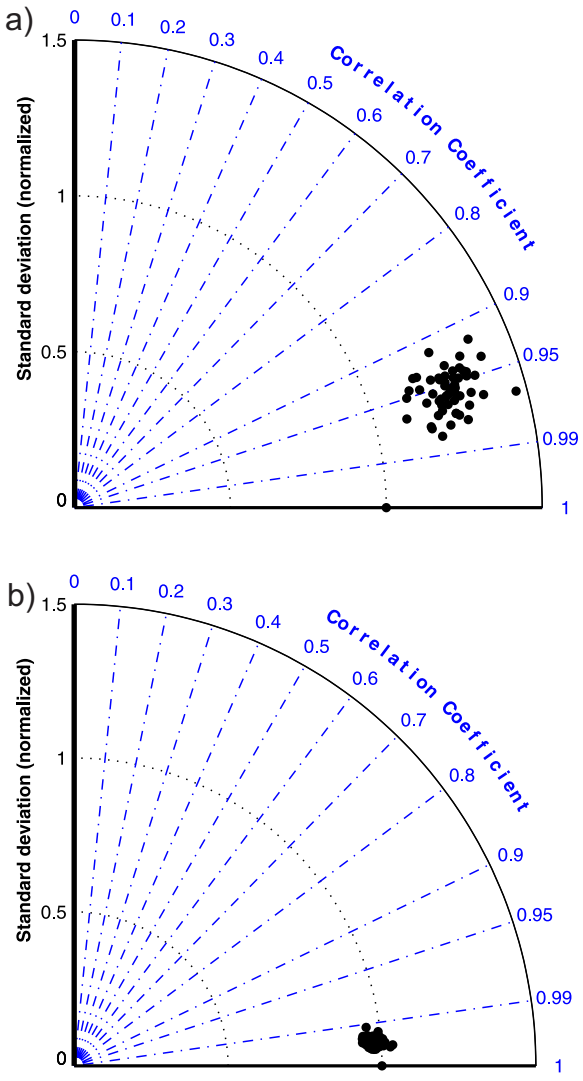

Figure 1: Taylor diagram comparison of the perturbed physics ensemble (Annan et al. 2005) versus the standard configuration of GENIE (RIdgwell et al. 2007) used for the synthetic data for (a) salinity and (b) temperature.

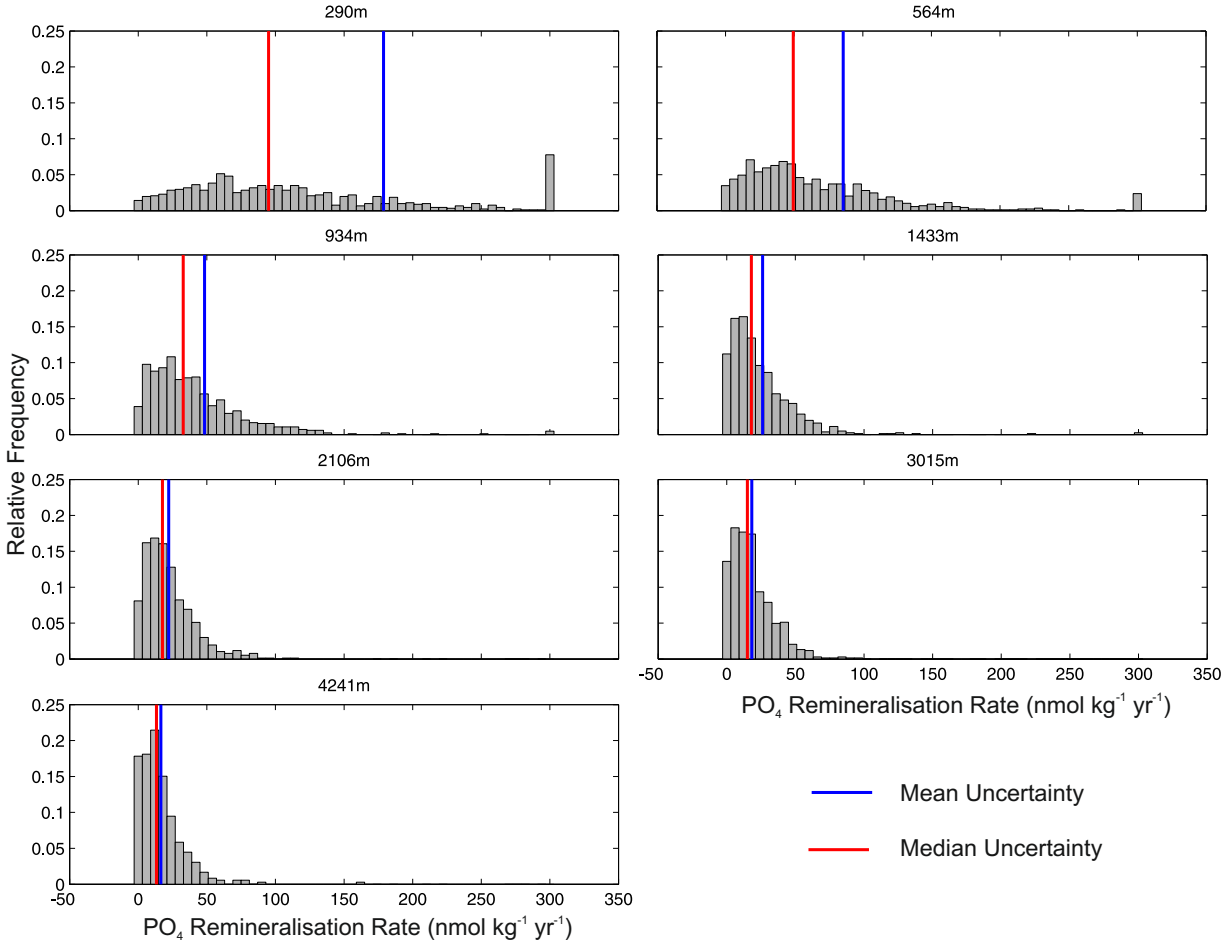

Figure 2: Histograms showing the distribution, mean and median standard deviation of estimated PO_4 remineralisation rates when simulating uncertainty associated with observations as shown in Figure 8 for each depth level in GENIE. The histograms show the distribution standard deviations calculated for all grid-boxes at that depth level when observation uncertainty is simulated. The blue and red lines indicate the mean and median respectively corresponding with the values shown in Figure 8. Histograms are shown with a relative frequency as the number of samples is different between depth levels. Values > 300 nmol kg^{-1} yr^{-1} are included in the last bin.

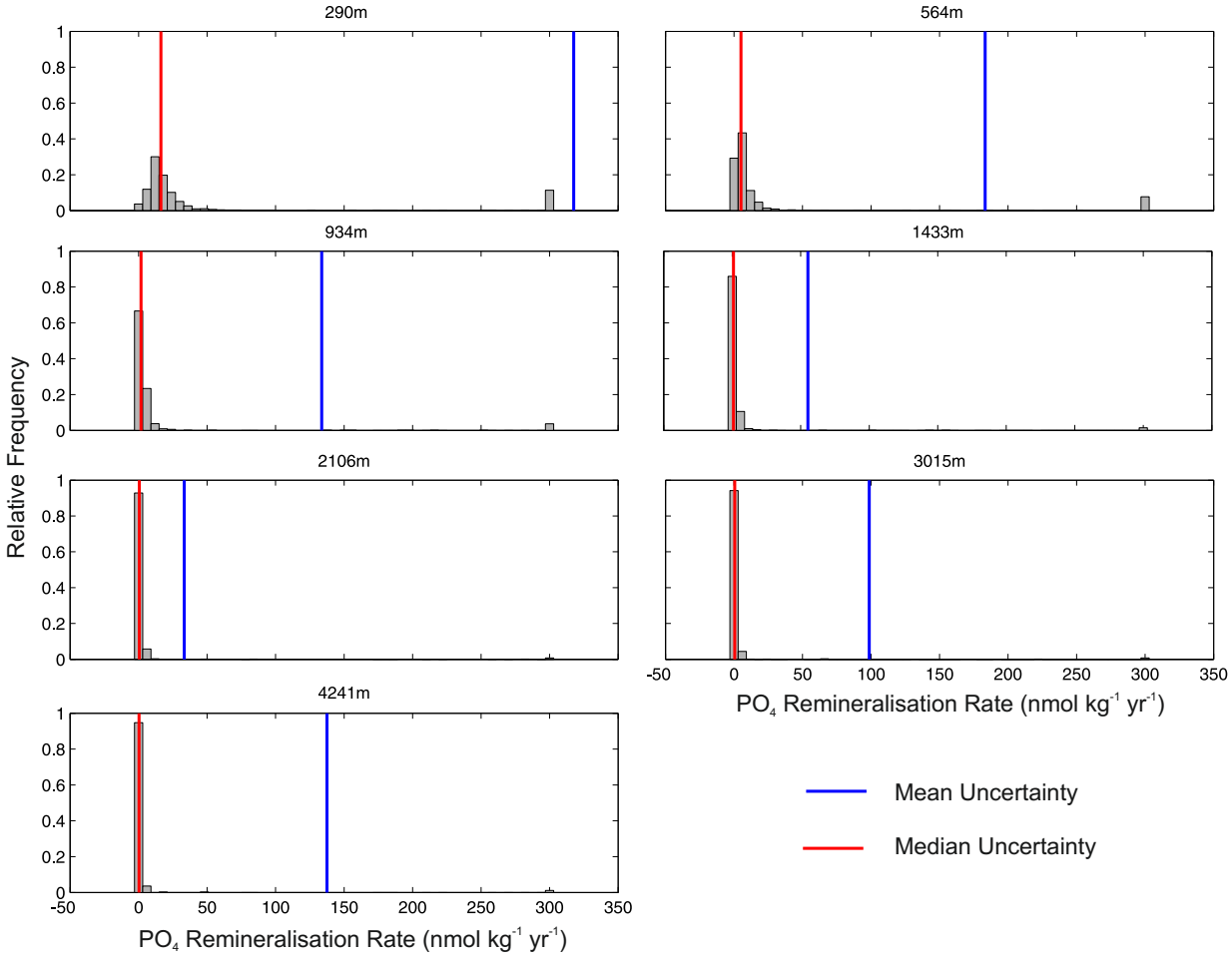

Figure 3: As per Figure 2 in supplementary material but for uncertainty associated with circulation. Note the figures are comparable except for the change in relative frequency magnitude shown.

References

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